Back Paper

1. (a) If P, Q have continuous partial derivatives in D and $\int P\,dx + Q\,dy$ is independent of paths in D then $\frac{\partial P}{\partial y} = \frac{\partial Q}{\partial x}$ in D.

(b) If D is simply connected and $\frac{\partial P}{\partial y} = \frac{\partial Q}{\partial x}$ in D then $\int P\,dx + Q\,dy$ is independent of paths in D.

2. (a) Give an example of a sequence of cts fns f_n defined on an interval $[a,b]$ such that $f_n \to f$ pointwise but not uniformly.

(b) Let $f_n \in C([a,b])$. If $f_n \to f$ uniformly then prove that $\int f_n\,dx \to \int_a^b f\,dx$.

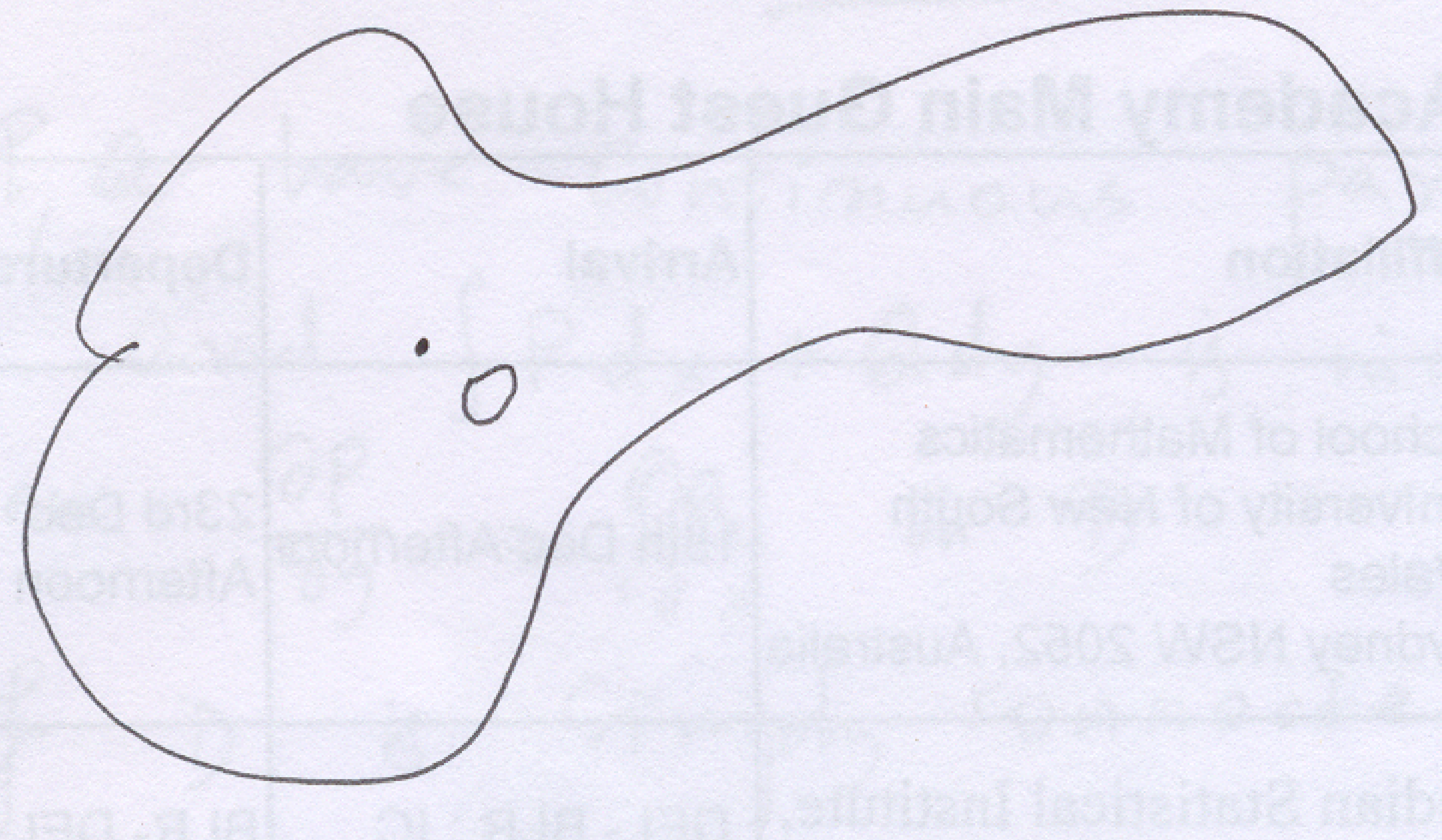
(c) Give an example to show that if we only assume that $f_n \to f$ pointwise then the conclusion above may no longer be valid.

3. A square integrable ~~f~~ 2π-periodic function f has Fourier coefficients given by

$$\hat{f}(n) = \begin{cases} \dfrac{1}{n^{99}} & n > 0 \\ e^{n/2} & n \leq 0. \end{cases}$$

Can f be a C^∞ function? Justify your answer.

4. Let $F(x,y) = \left(\frac{-y}{x^2+y^2}, \frac{x}{x^2+y^2}\right)$ be a vector field on $\mathbb{R}^2 \setminus \{0\}$. Find the integral of F over the path drawn below.



5. (a) Let $\nabla^2 = \nabla \cdot \nabla = \left(\frac{\partial}{\partial x}\right)^2 + \left(\frac{\partial}{\partial y}\right)^2 + \left(\frac{\partial}{\partial z}\right)^2$.

A function f is said to be harmonic if $\nabla^2 f = 0$. If f is harmonic, prove that div grad $f = 0$

(b) Let $f(x,y,z) = \dfrac{q}{4\pi\sqrt{x^2+y^2+z^2}}$.

Compute the integral of $E = -\text{grad}\, f$ over the unit sphere centred at the origin, that is, the integral $\iint_S E \cdot n \, d\sigma$